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NPIC/TP-22/63
August 1963

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TECHNICAL PUBLICATION

DETERMINATION OF PITCH AND ROLL FROM HORIZON EXPOSURES ON KH-4 SATELLITE PHOTOGRAPHY

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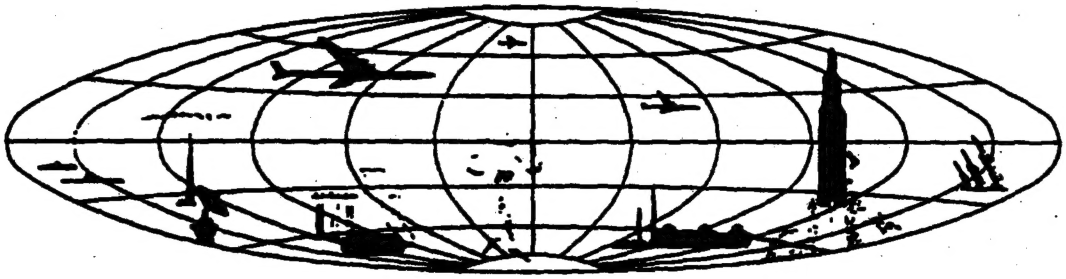

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NPIC/TP-22/63

DETERMINATION OF PITCH AND ROLL FROM HORIZON EXPOSURES ON KH-4 SATELLITE PHOTOGRAPHY

INTRODUCTION

Horizon photographs are used to determine the pitch and roll components of the KH-4 vehicle's attitude or, more properly, of an orbiting camera's attitude. Horizon photographs do not give any yaw information in cases where the primary or pan camera is aimed at the nadir (Figure 1). The determination of pitch and roll is simple and straightforward, and good approximations of both pitch and roll are easily obtained from simple measurements and computation.* When the primary camera is not oriented toward the nadir and the horizon cameras are attached to the primary camera so that the horizon photos have an apparent swing, as is the case with KH-4, the measurements are restricted to a smaller section of the horizon image and the data reduction becomes more complicated (Figure 2).

Many reduction systems have been proposed and used. They range from the extremely simple ones where simple scales are used to measure the image and the computation is done graphically, to the complex ones where precise machines are used to measure the image and the computation is accomplished with a computer. The following is presented as a solution falling between the extremes, but definitely toward the more complex approach. This determination of pitch and roll from horizon photographs is tailored to be practical for the reduction of a large number of photographs. It is assumed that measurements are made with a comparator, such as a Mann instrument, and that the computation is performed by a computer.

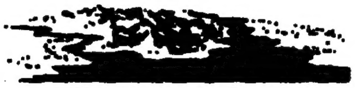

FIGURE 1. HORIZON IMAGE FOR VERTICALLY ORIENTED PRIMARY CAMERA.

FIGURE 2. HORIZON IMAGE FOR PITCHED PRIMARY CAMERA.

DEFINITIONS

Yaw, pitch, and roll are taken as orthogonal rotations about the three mutually perpendicular primary photo axes having their origin at the lens (Figure 3). Yaw is considered primary, pitch secondary, and roll tertiary. Since a mechanical gimbal system is not involved, the ordering of the orientation rotations is arbitrary though necessary to adequately define them. Swing, S, is a rotation of the x-y-axes about the z-axis in the plane of the photograph. Swing is positive when the x-y-axes are rotated to the left as seen from a point on the positive z-axis.

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*The final result is not error free, a significant error arises in the identification and measurement of the horizon image.

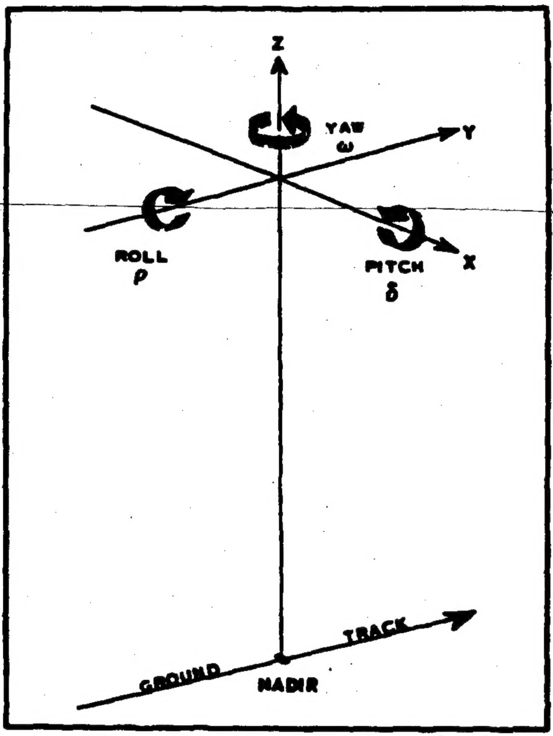

FIGURE 3. ATTITUDE COMPONENTS.

MEASUREMENTS

The measurement of image points is done with respect to an axis system on the photograph having its origin at the principal point. Ten points are measured on each horizon frame, as shown in Figures 4 and 5. The 4 fiducial marks are read first and then 6 points on the horizon image. Points 5 and 10 should be chosen as far apart as possible, and points 6 through 9 should be approximately evenly spaced between them.

COMPUTATION

Assume the following photo measurements:

$P_1(x_1, y_1)$ - end fiducial mark
$P_2(x_2, y_2)$ side fiducial mark
$P_3(x_3, y_3)$ - opposite end fiducial mark
$P_4(x_4, y_4)$ - opposite side fiducial mark
P_5 through P_{10} - approximately evenly spaced points on the horizon image.

The principal point is determined from the intersection of two straight lines. Instrument tolerances in placing the fiducial marks prohibit the method of taking the average coordinates of opposite fiducial marks. From the equation of a straight line:

$$y = x\left(\frac{y_4 - y_2}{x_4 - x_2}\right) - x_2\left(\frac{y_4 - y_2}{x_4 - x_2}\right) + y_2$$

and

$$y = x\left(\frac{y_3 - y_1}{x_3 - x_1}\right) - x_1\left(\frac{y_3 - y_1}{x_3 - x_1}\right) + y_1$$

These equations represent the fiducial lines defined by the opposite fiducial marks. Their simultaneous solution gives the coordinates of the principal point. It is convenient at this step, but not necessary, to reduce the coordinates of the points on the horizon curve to the principal point origin, i.e.: let

$$x'_5 = x_5 - x_p, \qquad \text{and} \qquad y'_5 = y_5 - y_p,$$
$$\cdots\cdots \qquad\qquad \cdots\cdots$$
$$x'_{10} = x_{10} - x_p, \qquad\qquad y'_{10} = y_{10} - y_p.$$

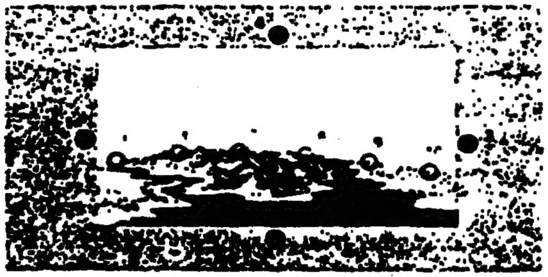

FIGURE 4 MEASUREMENT POINTS.

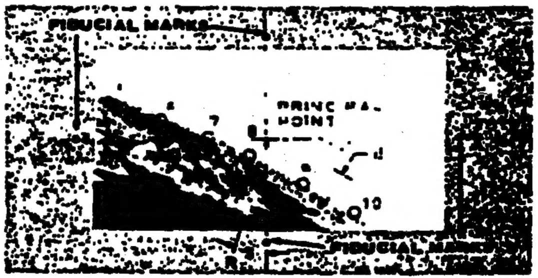

FIGURE 5. ROLL COMPONENT (d) AND SWING (S).

Assuming the horizon image to be circular, the center is found from the circle of best fit in the sense of least squares. It is worthwhile to note that the horizon image is not circular but, because such a small section of the actual hyperbolic curve is given, there is no practical difference. It is convenient to find the center of the circle. From the general equation of a circle:

$$x^2 + y^2 + Ax + By + C = 0.$$

There are six observation equations of the form

$$A\left(\frac{x_i}{x_i^2 + y_i^2}\right) + B\left(\frac{y_i}{x_i^2 + y_i^2}\right) + C\left(\frac{1}{x_i^2 + y_i^2}\right) + 1 = V_i,\ 5 \le i \le 10.$$

which give the following normal equations:

$$\begin{array}{ccccccccc} A & & B & & C & & K & & \\ \left[\frac{x^2}{(x^2+y^2)^2}\right] & + & \left[\frac{xy}{(x^2+y^2)^2}\right] & + & \left[\frac{x}{(x^2+y^2)^2}\right] & + & \left[\frac{x}{x^2+y^2}\right] & = & 0. \\ \left[\frac{xy}{(x^2+y^2)^2}\right] & + & \left[\frac{y^2}{(x^2+y^2)^2}\right] & + & \left[\frac{y}{(x^2+y^2)^2}\right] & + & \left[\frac{y}{x^2+y^2}\right] & = & 0. \end{array}$$

and

$$\left[\frac{x}{(x^2+y^2)^2}\right] + \left[\frac{y}{(x^2+y^2)^2}\right] + \left[\frac{1}{(x^2+y^2)^2}\right] + \left[\frac{1}{x^2+y^2}\right] = 0.$$

The simultaneous solution of these normal equations gives the coefficients of the equation of the circle. Then the center coordinates of the circle are:

$$x_c = -\frac{A}{2} \quad \text{and} \quad y_c = -\frac{B}{2}$$

The roll component, d, on the photo is merely the difference between the radius of the circle and the distance from the principal point to the center of the circle (Figure 5):

$$d = \tfrac{1}{2}\left\{\sqrt{A^2 + B^2} - \sqrt{A^2 + B^2 - 4C}\right\}.$$

The roll component ρ'', for the horizon camera is determined by:

$$\rho'' = \tan^{-1}\frac{d}{f}.$$

where f is the calibrated focal length of the horizon camera.

Swing is given by:

$$S' = \tan^{-1}\frac{B}{A}.$$

Because the y-axes of the horizon camera and the primary camera are not exactly parallel, the calibration data are used to rotate the horizon camera axes to parallel those of the primary camera. This rotation is significant in swing. The correction to the swing is

$$\Delta S = \tan^{-1}\frac{N-K}{D}.$$

where D, N, and K are calibration numbers given in the KH-4 calibration data for the port horizon camera of the forward set. Then

$$S = S' + \Delta S$$

The computation thus far is computed for both the starboard and port horizon cameras and then the average roll component,

$$\rho''_m = \frac{\rho''_p + \rho''_s}{2}.$$

is used to compute the dip angle, D', for the port horizon camera measured in the principal plane. This operation begins an iterative loop which usually yields good results in three cycles. The projection of the preset or "built-in" dip, D, to the principal plane is

$$D' = \sin^{-1}(\sin D \cos \phi). \quad \dots\dots\dots (1)$$

where ϕ is replaced by S for the first approximation. Then the roll in the principal plane is

$$\rho' = D' + \rho''_m. \quad \dots\dots\dots\dots (2)$$

and the roll in the pitched plane is

$$\rho = \sin^{-1}\left(\frac{\sin \rho'}{\cos \phi}\right). \quad \dots\dots\dots (3)$$

where ϕ is replaced by S for the first approximation. The azimuth of the principal plane is computed from the expression

$$\alpha = \tan^{-1}(\tan \rho \sin \phi), \quad \dots\dots\dots (4)$$

where ϕ is replaced by S for the first approximation. Delta is really half the tilt of the horizon photo, therefore

$$\delta = \frac{90-\rho'}{2}. \quad \dots\dots\dots\dots (5)$$

which gives all of the components required to compute the pitch:

$$\phi = \tan^{-1}\left[(\tan S + \tan \alpha) \cos \alpha \tan \delta\right]. \quad \dots\dots (6)$$

New values for ρ, δ, α, and ϕ are computed using the last computed values in equations

(1) through (6). Three iterations are usually enough. The values of (equation 3) and (equation 6) refer to the left horizon photo.

The pitch and roll for the primary camera is determined by adding the calibration data. The KH-4 calibration data provides the following:

1. y_c - the y-coordinate of the pan camera calibration mark. Then the pitch for the pan camera is given by

$$\omega_1 = \omega + \tan^{-1}\frac{Y_1}{f} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

where f is the calibrated focal length of the pan camera. In the case of roll, the KH-4 calibration data is:

2. x_h the x-coordinate of the horizon camera calibration mark, which is co-planar with

3. x_c - the x-coordinate of the pan camera to the corresponding mark.

Then the roll for the pan camera is

$$\rho_p = \rho + 90 + \tan^{-1}\frac{X_h}{f} + \frac{180\, X_{hc}}{\pi f}$$

$$= \rho + 15 + \tan^{-1}\frac{X_h}{f} + \frac{180\, X_{hc}}{\pi f} \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

If only one horizon photo is used, the altitude is used in the computation. In this case ρ is computed as above, and then tilt is determined from

$$T = \sin^{-1}\frac{R}{R+H} \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (9)$$

where R is the mean radius of the earth and H is the altitude above the surface. The insensitivity of this solution indicates that the radius approximation is valid. Then

$$\delta = \frac{T}{2}$$

and

$$\rho' = \frac{1}{2} + \rho'$$

Now the three-cycle iteration with equations (1) through (6) will yield the pitch and roll for the horizon camera.

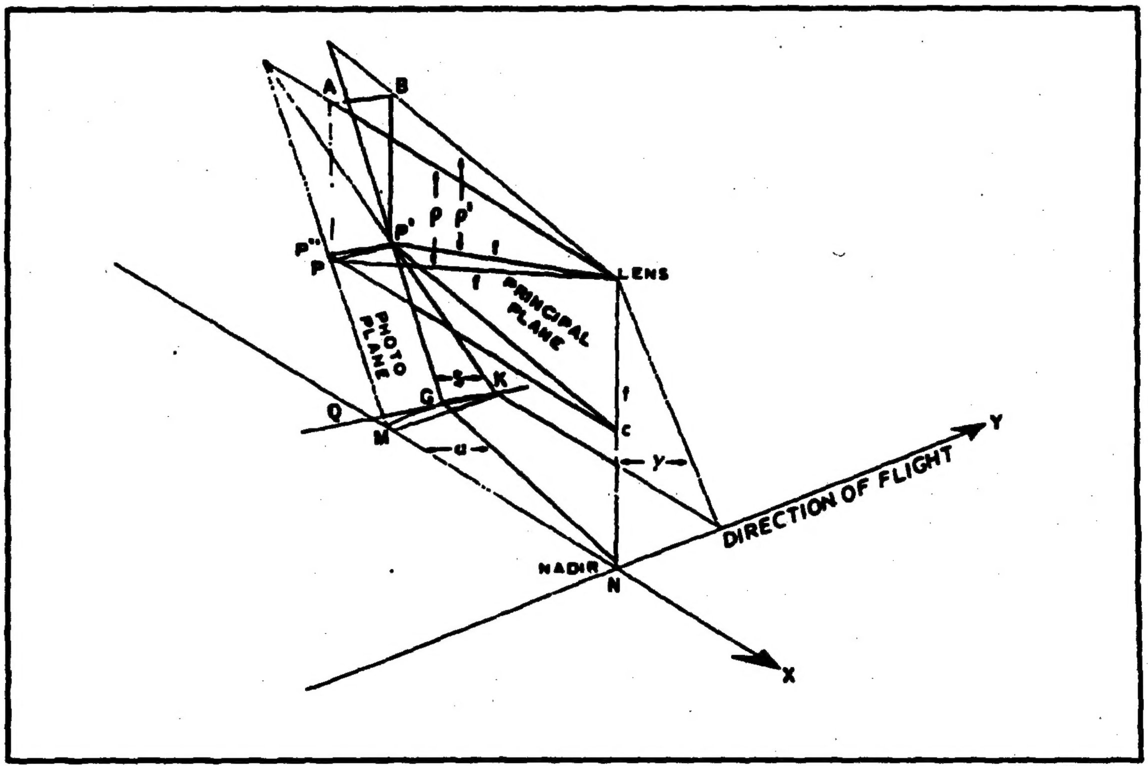

FIGURE 6. GEOMETRY OF TILTED HORIZON PHOTO.

APPENDIX

Derivation of Equations 3, 4, and 6 (Figure 6).

1. Equation (3):

$$\sin \rho' = \frac{P'B}{f} = \frac{f \sin \rho \cos \phi}{f}.$$

$$\rho = \sin^{-1}\left(\frac{\sin \rho'}{\cos \phi}\right).$$

2. Equation (4):

$$\tan \alpha = \frac{P'P''}{P''C'} = \frac{f \sin \rho \sin \phi}{f \cos \rho}.$$

$$\alpha = \tan^{-1} (\tan \rho \sin \phi).$$

3. Equation (6):

$$\tan S = \frac{GK}{GP'} = \frac{KQ-GQ}{GP'}.$$

$$KQ = KM \sec \alpha = f \tan \phi \sec \alpha.$$

$$GQ = GN \tan \alpha = f \tan \delta \tan \alpha.$$

$$\tan S = \frac{f \tan \phi \sec \alpha - f \tan \delta \tan \alpha}{f \tan \delta}.$$

Rearranging:

$$\tan \phi = \frac{\tan \delta \tan S + \tan \delta \tan \alpha}{\sec \alpha},$$

$$\phi = \tan^{-1}\Big[(\tan S + \tan \alpha) \cos \alpha \tan \delta\Big].$$

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